Illustrating the History of the Planimeter

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Abstract

An analogue device that can perform mathematical integration, the planimeter was first invented in Bavaria in 1814. Throughout the nineteenth century it was developed in a variety of ways, receiving many of its improvements from famous scientific names such as Lord Kelvin and James Maxwell. Later it would become a central component of famous machines like Kelvin’s harmonic analyser and the differential analysers of the early twentieth century.

This project traces the history of this significant but often forgotten device; and ‘illustrates’ its development with a number of computer-based interactive environments.

Key words

History of Computing
History of Mathematics
Empirical Modelling
Planimeter
Analogue Computing
Differential Analyser
Mathematical Instruments
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Author’s Assessment of Project

Technical contribution

This project contributes to the empirical modelling knowledge-base. It illustrates how an analogue device can be modelled on a digital platform. Modelling a physical mechanism like the planimeter is a superb example of dependency in the real-world and in this way, my models will serve as useful examples of modelling physical devices.

Achievements

The history of planimeters is relevant as a part of the wider history of analogue computation. The main achievement of the project has been the bringing together of sources that do not exist together in English in any other major publication. The project has also successfully demonstrated how a project can be a mixture of both software and historical enquiry.

Weaknesses

Issues of how the models should be interacted with were not fully addressed due to time constraints. The project would be far stronger if there was more information.

There should also be more detail about the modelling process as a form of learning about history. Again this is an issue that has been pushed aside due to the bulk of material emerging from the historical research.
Acknowledgements

I would like to take this opportunity to formally thank Steve Russ for all his hard work throughout the life of the project.

Thanks also go to Meurig Beynon and Ashley Ward and the other empirical modelling students for their help with the Eden tools. Rachel Bowers and Tom Swindells for their endless support and proof-reading. Thanks to Chris Burland for help with .obj files.

A number of images in this project have been lifted from external sources. These are cited in the usual way from within the image caption. If there is no citation on a graphic it is the work of the author.
Chapter 1

Introduction

“The most common form of mathematical instrument available in the nineteenth century was the planimeter.”

[Cro90]

Planimeters are instruments for measuring the area of figures drawn or projected onto a surface. Treated independently, they are merely elaborate measuring devices. But the planimeter provided a principle of mechanical integration that was central in developing more complex devices. The acme of this development was when the differential analyser was constructed in 1927.

The history of science is scattered with examples of principles developed independently by different people. Often it is the case that the inventions occur together after an extended period of no development. The invention of the calculus is an example of this phenomenon. Something in the academic climate of the seventeenth century demanded tools to solve certain types of problem and this climate led to both Leibniz and Newton discovering equivalent methods.

The history of the development of the planimeter can be thought of in much the same way. There was a growing demand for area calculation, and the numerical methods available were not satisfactory. The early 1800’s provided just the correct climate for the development of the planimeter to flourish, possibly due to increasing reliability of mechanical systems. This period saw the principle of the planimeter invented and re-invented a number of times.
CHAPTER 1. INTRODUCTION

1.1 Applications of the planimeter

1.1.1 Area measurement

It was estimated that in the 1850’s there were over six billion land areas requiring evaluation in Europe [Hen94]. All the early developments of the planimeter were driven by the need to calculate area and particularly land area. The first inventor was a land surveyor (see Chapter 4) and another (see Chapter 8) was used by the Ordinance Survey in Scotland who observed that the device had “the peculiar advantage, that it gives the areas of the most irregular figures with the same accuracy that it gives the areas of the most regular, and with the same facility” [San52, page 124].

1.1.2 Indicator diagrams

The second major use of planimeters was in the analysis of steam engines, where it was necessary to find the area of an indicator diagram. An indicator diagram represents the displacement of a piston against the pressure in that piston [Cas51] which is measured by a small device known as the indicator [www00c]. The area of such a diagram is proportional to the force delivered and provides a way of comparing the power output of different engines.

This became a widespread use for planimeters and even lead to the development of planimeters with specific scales. An example of this is the Willis planimeter [Sch93] that had a special “horse-power attachment” so that it could directly give a measurement of the power delivered by an engine.

Another instrument of interest is the dynamometer which measures force. Dynamometers – which were also mechanical integrators – could be connected directly to the steam engine.

1.1.3 Integration

Integration is the mathematical method of finding the area bound by that function when plotted. It is entirely appropriate that planimeters should be used as mechanical integra-
1.2 Motivations of the project

It is their use in the differential analyser that gives the planimeter a significant place in computing history and this project is written from that perspective. The project was motivated by a desire to know more about how mechanical integrators – one of the principle components of analogue computing – first evolved.

Throughout this research, emphasis has been placed on the devices that relate to the differential analyser in this way.
Part I

The History of the Planimeter
Chapter 2

Landmarks in the Development of the Planimeter

2.1 Key participants in planimeter history

This chapter aims to provide an overview of the key people who contributed to the design of the planimeter. These participants range from original inventors and manufacturers through to those who employed the principles in larger machines.

2.1.1 First inventors

There is evidence to suggest that the planimeter was invented by a number of ‘first inventors’ who developed their ideas in isolation.¹

Johann Martin Hermann

The invention of the planimeter is generally attributed to Hermann, working in Bavaria in 1814 [Bax29a]. Fifer refines this location further to Munich and confirms the date as 1814 [Fif61b].²

¹Bromley [Bro90] refers to four separate discoveries by Hermann, Gonnella, Oppikofer and Sang.
²However, in Volume 1 of the same work he gives the later date of 1819. [Fif61a].
Hermann’s planimeter used what would later be known as a cone-and-disc mechanism (see Chapter 3).

**Lammle**

Although most sources attribute the early Bavarian work to Hermann, a few mention some kind of collaborative work between Hermann and Lammle.³

**Tito Gonella**

A further re-invention is attributed to Gonella [Bro90] who was working in Florence in 1824 [CU14]. Gonella’s device used a disc rolling on a sliding cone.⁴ Later, Gonella would realised that this cone could be replaced with a disc [CU14].

**Johannes Oppikofer**

Working in Switzerland, Oppikofer⁵ also used a cone as the basic component of his planimeter in 1827 [CU14]. His design was manufactured in France by Ernst around 1836 [Bro90] and later by Clair [Fis95].

There seems to be widespread knowledge of Oppikofer. Ocagne [Oca28] attributes the first discovery of a wheel rolling on a disc or a cone to him, and his planimeter is the first to be recorded in Morin’s Les Appareils d’Intégration [dM13].

**John Sang**

Sang’s ‘platometer’⁶, rolls on the surface of the area being measured much like the later designs of Coradi. It also uses a cone to perform the integration. Sang exhibited his invention at the Great Exhibition in 1851 and presented a formal paper to the Royal

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³Fifer[Fif61b] refers to joint development at the University of Munich. Carse and Urquhart[CU14] associate Lammle with improvements to Hermann’s design.
⁴A superb illustration of this principle is provided on a German Website. See [Vol00].
⁵Ocagne identifies him as Hoppikoffer [Oca28] but Carse and Urquhart [CU14] and more recent literature favour the spelling Oppikofer. Fischer [Fis04] stands by the correct rendering being Oppikofer.
⁶See note 9.
2.1. KEY PARTICIPANTS IN PLANIMETER HISTORY

Scottish Society of Arts in early 1852 [San52]. Here is perhaps a clear example of re-invention; Sang doesn’t call his device by the standard name and responses from the society to his paper suggest that his ideas were received as fresh.

Sang can be found in the Directory of British Scientific Instrument Makers with the following description:

<table>
<thead>
<tr>
<th>Name</th>
<th>Occupation</th>
<th>Address</th>
<th>Ex year</th>
<th>Known to have sold:</th>
</tr>
</thead>
<tbody>
<tr>
<td>SANG John</td>
<td>Manufacturer</td>
<td>Kirkcaldy, Fife</td>
<td>1851</td>
<td>planimeter</td>
</tr>
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</table>

Sources: cat. 1851 [Cli95]

Sang was from Kirkcaldy in Scotland and related to a Mr Edward Sang of Kirkcaldy, a famed engineer and scientist of the same period [Cra97].

Jacob Amsler

Amsler developed a planimeter based on a different principle, known as the polar planimeter. A large number of polar planimeters were manufactured and are still available today (see Section 2.4).

2.1.2 Development and manufacture

Poncelet & Morin

Fifer [Fif61b] refers to these as being based in France, improving on Hermann and Lammle’s designs in the 1830’s.⁷ They were applied mathematicians in Metz, France. They had both served in the French military as engineers and made many improvements to turbines and water wheels. Morin received the honour of being one of the 72 prominent French scientists to be commemorated on the plaques of Paris’ Eiffel Tower [OR96a].

⁷Fifer actually refers to “Poncelet and Morin” but it is assumed that this is a misspelling for Poncelet.
CHAPTER 2. LANDMARKS IN THE DEVELOPMENT OF THE PLANIMETER

**Heinrich Rudolf Ernst**

Based in Paris, Ernst manufactured Oppikofer’s design in 1836 [Bro90].

**Kaspar Wetli**

In 1948, Wetli in Zurich made the design decision to replace the cone in the planimeters he knew with a disc. This refinement allowed Wetli to integrate negative-value functions [Bro90].

**Georg Christoph Starke**

Starke manufactured Wetli planimeters in Vienna [Bro90]. Developments lead to the Wetli-Starke planimeters.

**Hansen**

Based in Gotha (in Bavaria), Hansen improved on the Starke planimeters and manufactured with Ausfeld [Bro90].

### 2.1.3 Laying the foundations of the differential analyser

**James Clerk Maxwell**

The famed physicist observed how important the function of the planimeter was.

> “The measurement of the area of a plane figure is an operation so frequently occurring in practice that any method by which it may be easily and quickly performed is deserving of attention.”

[Max55b]

Maxwell had seen Sang’s platometer at the Great Exhibition and had been “greatly excited” [Max55b] by it. His interest in the device was more academic than practical and his own platometer was never manufactured.\(^8\) Using a sphere, his contribution was

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\(^8\)Most sources make this claim, although Maxwell certainly investigated the possibility of having the platometer constructed. See Chapter 9.
to eliminate errors introduced through the integrating wheel not making perfect rolling contact.

James Thomson

The brother of Lord Kelvin, Thomson used Maxwell’s principle of pure rolling and devised a mechanism that used a sphere rolling on a disc and a cylinder [Tho76a].

The disc-ball-cylinder integrator still didn’t completely avoid slipping and in a letter to Thomson, Maxwell suggested further ways of improving the design [Max79].

Lord Kelvin (Sir William Thomson)

Thomson and the other contributors had seen the planimeter as a stand-alone device. What revolutionised matters was when Kelvin realised that the disc-ball-cylinder integrator could be used as a component in larger machines like the harmonic analyser and the tide predictor. He realised that linking the integrators together could allow the solution of differential equations to be found mechanically [Bro90].

Vannevar Bush

Although Kelvin understood how integrating machines could be arranged to form a computer capable of solving differential equations, such a device wasn’t constructed until Vannevar Bush built the differential analyser in 1927 [Cra47]. The central problem was that the output from one of Thomson’s integrators was not sufficiently powerful to act as the input to another. The solution to this problem was the torque amplifier developed by Nieman, an engineer working with Bush.

2.2 Planimeters at the Great Exhibition

In 1851, The Great Exhibition of the works of Industry of all Nations was opened in the purpose built Crystal Palace in Kensington, London. Exhibits from all over the world were on display in 30 classes. Among them were a small number of planimeters from various
European inventors. These were on display under class X (philosophical instruments). The exhibitors were Sang, Gonella, Laur, Wetli and Ausfeld. Sang, who was the only English contributer, exhibited his ‘planometer’. The device was based on a rolling cone whereas the other planimeters exhibited used a disc. The exception to this was Laur’s “Olrithme” which was not a mechanical planimeter in the conventional sense but rather an instrument that was based on the triangulation of a plane [GE52, p304].

These exhibits achieved notable respect. All the exhibitors of planimeters were awarded “honourable mention” and Gonella attained a “Council Medal” [GE52, p304].

2.3 Early histories of the planimeter

Before developing the history of these people and their inventions further, it will be helpful to look at three major histories of planimeters written in the late nineteenth and early twentieth centuries. These are principle sources of the following material, but also interesting as history themselves. There are of course other early collections of information about these devices; one such example is Bauenfeind’s article in the German Dingler’s Journal. Of the following three the youngest two have been the most useful in the preparation of this project; the earlier source is only a recent addition to my bibliography but has clarified many points.

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9Later that year, Sang would rename his device the ‘platometer’ but never adopt the more standard name ‘planimeter’.
10Jean Antoine Laur, a Parisian manufacturer who exhibited philosophical instruments [GE51, p1205].
11See Chapter 4, note 1.
2.4. CURRENT MANUFACTURE

2.3.1 Henrici

Henrici’s article [Hen94] was commissioned by the Royal Society for the Advancement of Science and offers a very complete description of both the types of planimeters and their history. Particularly interesting is Henrici’s ability to comment on the Great Exhibition. With the benefit of twenty years perspective, Henrici is able to reflect while still in touch with many primary accounts. He attempts to correct and harmonise the Jury Reports of the Great Exhibition. For example, it is Henrici who links Dr Flaussen of Seeberg (see Chapter 7) with Wetli and therefore illustrates a link between two of the Crystal Palace exhibitors that the Great Exhibition literature does not record.

2.3.2 Morin

Henri de Morin should not be confused with the Arthur Morin who worked with Poncelet (see Section 2.1.2) on the dyanometer, another mechanical integrator.

Morin’s work, written in France in 1913, is a monograph illustrating the various mechanical integrators known to him at that time. *Les Appareils d’Intégration* [dM13] provides an excellent source of otherwise unavailable diagrams.

2.3.3 Carse and Urquart

Published in the handbook of the Napier tercentenary celebrations [Hor14] are a collection of articles by these two authors. One of these is an article on planimeters [CU14]. Written a year later than *Les Appareils d’Intégration*, the article cites Morin heavily and also cites Henrici’s article. In terms of new information the article is fairly limited, but like Morin, Carse and Urquart provide a vast number of illustrations, this time of polar planimeters.

2.4 Current manufacture

Planimeters are still available (and presumably in industrial use) today. On the web I have discovered two modern manufacturers who produce planimeters, *Gebrüder HAFF*
CHAPTER 2. LANDMARKS IN THE DEVELOPMENT OF THE PLANIMETER

“founded in 1835, for the manufacture of mathematical instruments” [www00b] and Tamaya Technics, a modern Japanese instrument company.

The planimeters produced by HAFF are mainly mechanical and have changed very little in construction from the devices produced in the late nineteenth century. Both companies also construct high-end versions marketed as ‘digital’ planimeters. These digital machines still consist of the analogue technology but include a digital meter that records motion of the integrating wheel with precision.

The planimeters are precision instruments, trading for between 200 (for a mechanical version) and 500 Euros (for a digital version).

On their website, HAFF supply a list of around 50 common uses for planimeters. These include botany (area of leaves), medicine (cross-sectional area of organs) and surveying (land areas) [www00b].
Chapter 3

Themes and Ideas

3.1 Mechanical integration

3.1.1 A mathematical explanation

The wheel-and-disc integrator gives a clear diagram of the principles used in a mechanical integrator. Both the small wheel and the larger disc are free to rotate on their axes.

The axis of the disc is fixed but the position of the wheel can change to allow the distance, $\rho$ of the wheel from the centre of the disc to be varied. The wheel sits on the surface of the disc so that it rolls as the disc rotates.

Let us consider a rotation of the disc; in Figure 3.3 the disc has rotated by $\theta$ radians.
During this rotation of the disc, the wheel will have rolled over the surface of the disc. We define its rotation as $\phi$ radians.

Now it should be clear that the distance, $d$, travelled by the wheel over the surface of the disc is equal to the portion of the wheel’s circumference subtended by $\phi$ radians.

$$d = r\phi$$

And since $d$ is also the length of the arc traced on the surface of the disc.

$$d = \rho \theta$$

$$\phi = \frac{\rho \theta}{r}$$

When integrating a function $f(x)$, changes of the disc rotation correspond to changes in $x$, and changes of the displacement of the wheel from the centre of the disc correspond.
to changes in \( f(x) \). It is therefore possible to say that \( \theta \) was a small change in \( x \), \( \delta x \) and that \( \rho \) is \( f(x) \). Hence we get:

\[
\phi = \frac{1}{r} \delta x f(x)
\]

which as \( \delta x \to 0 \) becomes:

\[
\phi = \frac{1}{r} \int f(x) dx
\]

Hence, the rotation of the integrating wheel is proportional to the integral of \( f(x) \).

\[
\phi \propto \int f(x) dx
\]

### 3.1.2 A mechanical explanation

In order to find the area under a curve, it is common to take the sum of small strips of area. This is a digital approach to the problem, although in the calculus the strips become infinitely narrow. The idea of infinitely thin strips is a very difficult concept to grasp and use whereas a planimeter provides an elegant analogue solution to a continuous problem.

![Figure 3.4: As \( \delta x \) becomes small the sum of the strips approximates the integral](image-url)
CHAPTER 3. THEMES AND IDEAS

Integrating a constant function

Finding the area under a constant function is trivial but is the first step in understanding mechanical integration. The solution can be found by rolling a wheel along the function plot. This is shown in Figure 3.5

![Figure 3.5: A roller can be used to calculate the length $S$](image)

The rotational displacement of this roller will derive a value for the length $S$. The area under this line is determined by the multiplication of $S$ and $h$.

Integrating a two-state function

Integrating a constant function is fairly trivial and finding the area of a two-state function is much the same. We can see from Figure 3.6 that this is just the same as doing the simple area calculation for each 'block' of the function.

However, in this example we’ll be looking at whether the area can be calculated in a single sweep with a modified form of roller.

To find the area under the function in 3.6 we need to know:

- The length of travel of the roller between position 1 and position 2. This is marked on the diagram as $S_1$.
- The length of travel of the roller between position 2 and position 3. This is marked on the diagram as $S_2$.
- The height of the function in its 'high' state. This is marked on the diagram as $h_1$. 
3.1. MECHANICAL INTEGRATION

- The height of the function in its 'low' state. This is marked on the diagram as $h_2$.

Formally, this is:

$$\int f(x)dx = S_1 h_1 + S_2 h_2$$

We can measure $S_1$ and $S_2$ by the rotation of the rolling wheel. The roller mechanism can also give the values of $h_1$ and $h_2$ (see Figure 3.7) if a means for providing vertical displacement is available.

Figure 3.6: Calculating the area under a two-state function

Figure 3.7: The displacement of the end of the rollers shaft gives the height, $h$. 
Figure 3.8: Scaling the rotation according to the function’s height.

**Integrating a Continuous Function**

A further development on this is to have two gears that mesh with a cog at the end of the roller’s shaft. These gears (see Figure 3.8) are sized and positioned to provide the multiplication by $h_1$ and $h_2$ respectively. The area is given by the sum of the rotations of both gear wheels.

The roller mechanism solves our continuity problem in the x direction but we have had to digitise the y values to enable the use of gear mechanisms.

To find the area under the function in Figure 3.5 using our existing technique would require us to have an infinite number of gears. What is necessary is for us to use a non-digitised gearing system or what is known as a *variable gear*. All the different mechanical planimeters and integrators have at their heart a variable gear, generally constructed from either a cone or a wheel.

### 3.2 The classifications of planimeter

Various authors have offered different ways of classifying the different planimeters. In the Handbook of the Napier Tercentenary Celebration, Carse and Urquart separate their planimeters into “rotating planimeters” and “planimeters with an arm of constant length.”

Henrici adopted a three way classification of orthogonal, polar co-ordinate and polar
3.2. THE CLASSIFICATIONS OF PLANI METER

Figure 3.9: The cone provides a different gear for every possible value.

(Amsler type). In this work, the latter two will be treated together.

3.2.1 Orthogonal planimeters

These planimeters are based around the idea of cartesian co-ordinates and therefore often consist of a carriage mounted on a track or heavy roller (providing an x displacement) and a pen mounted on the carriage that travels perpendicular to the track (providing a y displacement). It is these planimeters that developed into the integrator components of the differential analyser and so are the class that will get the most treatment in this report.

3.2.2 Polar planimeters

These are usually more of an area calculating instrument than the other type. Generally smaller and cheaper, polar planimeters were very popular towards the end of the nineteenth century as a means of calculating area. They tend to work by repeatedly tracing and un-tracing sectors of circles. The reason why they did not become integrating components in machines is because they were nearly exclusively of the rolling planimeter type
(that is without a track). Such devices really cannot make the transition from being a paper-top device to being an mechanical component. However, it is possible to build polar planimeters that can be controlled simply by shaft rotation, such a device was designed by Maxwell (see Section 9.3). Cartesian-based devices would be the choice for Lord Kelvin and Vannevar Bush in their analogue computers (see Chapter 11).
Chapter 4

Johann Martin Hermann

Hermann was a land surveyor [Fis95] in Bavaria during the early nineteenth century. His planimeter was developed for personal use in 1814 and he did not publish any of his work. The first publication to mention Hermann was a German article published in 1855\(^1\) and it is generally undisputed that he was the first to conceive of the idea. Of course, the fact that Hermann never published his work leads on to the question of how many other planimeters were privately invented.

Although most historical texts state that Hermann was the beginning of planimeter history, it is surprising how few actually give an adequate description of his mechanism. In an article published in the *Annals of the History of Computing*, Clymer [Cly93] offers a diagram of a “Hermann Integrator” but this description is in fact quite wrongly attributed and shows a wheel-and-disc mechanism of the kind adopted by the Richard Brothers (see Section 7.5). Of course it is possible that Hermann did develop a planimeter that used the wheel-and-disc principle, but such an advanced design would have been unnecessary for his purposes.\(^2\)

The actual device was scrapped in 1848 and the only surviving diagram of the planimeter is reproduced in Figure 4.1. As printed here (on A4 paper), the diagram is about three

---

\(^1\)Bauenfeind. *Zur Geschichte der Planimeter* in Dingler’s Polytechnische Journal 1855. It is referenced in both [Hen94] and [Fis95].

\(^2\)Wheels allow negative integration, but this is not required for calculating land areas.
quarters actual size.\(^3\)

This original diagram provides us with only one elevation of Hermann’s planimeter. The cone is vertical – this is unusual in planimeter design – and rotates in proportion to the sideways movement of the pen. How the pen affects displacement of the wheel is a little more subtle. As the pen moves forward and backward, the wheel’s carriage moves with it, this motion causes a small guiding wheel to move over a wedge. This guiding wheel is attached to the end of the shaft and that causes the position of the integrating wheel on the cone to vary (See Figure 4.2).

\section{4.1 Developments by Lammle}

Details of Lammle’s work are not clear, however one source dates his work to 1816, two years after Hermann’s first design and bases him in Munich [CU14]. Since Lammle is so infrequently reported in the historical accounts it should be satisfactory, at least at this stage, to make the assumption that Lammle’s development did not make significant changes to Hermann’s design. Perhaps his work related to small mechanical adjustments that improved the accuracy of the basic mechanism.

Fischer dates the manufacture of the planimeter as “c1817/18” [Fis04] and indicates that there was development going on “at least until 1819.” This development involved evaluating different materials that could be used for the cone. This links in with Fifer’s confusion over the date of invention being either 1814 [Fif61a] or 1819 [Fif61b].

It has already been said that Hermann’s work was fairly isolated from later developments. There is no evidence of a current publication of Hermann and Lammle’s work. The first articles on planimeters date from 1825 and relate to Gonnella’s work [CU14].

\footnote{In the actual planimeter the base of the cone had an 82mm diameter.}
Figure 4.1: Hermann's planimeter [Fis04]
The wheel rests on the wedge as the cone moves.

Figure 4.2: The wheel’s height is adjusted by a wedge as the cone moves.
Chapter 5

Tito Gonella

Professor Tito Gonella is the second known inventor and it is accepted scholarship that he developed his planimeter in complete ignorance of Hermann’s work [Hen94]. Unlike Hermann, Gonella published a description of his planimeter almost immediately [Fis95]. The story of Gonella’s instrument is of interest since he is the first to publish anything on the topic of planimeters.

Gonella worked at the University of Florence in Tuscany (Italy) [Fis95] and first conceived of a planimeter of the wheel-and-cone variety in 1824. He quickly realised that this principle generalised into what we know as the wheel-and-disc type. Henrici thought that Gonella only ever had one planimeter manufactured in Florence and that this was not the “well-executed piece of mechanism” [Hen94] that he required.

5.1 Swiss manufacture

In 1825, the archduke of Tuscany decided that a Gonella planimeter should be added to his personal collection. Gonella decided that the construction of his instrument required the precision of Swiss engineering and so he arranged for the design to be sent to a number of manufacturers in Switzerland. This course of action proved unsuccessful and Gonella did not “[succeed] in getting what he wanted” [Hen94].

A year after these designs were circulated, Oppikofer’s planimeter was invented in
Switzerland. Henrici has his suspicions that Oppikofer’s design was inspired by Gonella (see Chapter 6). However, the planimeter exhibited by Gonella at the Great Exhibition, was of the wheel-and-disc variety [GE52]. Oppikofer’s planimeter employed a cone as the integrating component and therefore could not have been a copy of this instrument. It is possible that Gonella sent out designs for both wheel-and-cone and wheel-and-disc planimeters to different manufacturers. There is no mention of Gonella utilising the principle of negative integration, certainly this would not have been on the agenda of a designer of a area-measuring instrument. Other possibilities are that Oppikofer did invent his planimeter independently\(^1\) or that Gonella actually had a cone planimeter manufactured and then later (before 1851) a disc mechanism. There is however, no evidence to support this. Rather, sources indicate that Gonella never actually constructed a cone planimeter.

Whether future developments were Gonella inspired or not, Gonella certainly contributed richly to this history. In him, the planimeter received proper academic treatment, subsequent publishing and found a home in the house of royalty.

\(^1\)Henrici does admit that it should be “quite in conformity with other instances in the history of science that several men should perfectly independently make the same invention.” [Hen94]
Chapter 6

Johannes Oppikofer

Bromley [Bro90] attributes “a further rediscovery” to Oppikofer. As we saw earlier, some sources have suggested that his discovery might not have been as original as Bromley suggests. In 1894 Henrici resigned himself that “How much he had heard of Gonnella’s invention or of Hermann’s cannot now be decided” [Hen94]. The earliest planimeter to be described in Morin’s work is Oppikofer’s. In turn Morin’s source is Ocagne [Oca28] from whom he also inherits the misspelling Hoppikofer.

This instrument works exactly like the principle described in Figure 3.9. Like Hermann’s it consists of a carriage that rolls along on a track, although motion along the track causes rotation of the cone rather than changing in the gear ratio. On the surface
of the cone is a wheel (R\textsuperscript{1} in Figure 6.1) that rolls when the cone rotates. This wheel is known as the integrating wheel. To provide the variable gear necessary for integration, the wheel can be positioned anywhere along the cone’s surface, from the vertex to the base. This position is linked to the sideways motion of the tracing point via a shaft (T\textsuperscript{2} in Figure 6.1) at the base of the carriage.

The area traced is given by the rotation of the integration wheel. This wheel drives two calibrated dials, one to the side of the integrating wheel and the other above it. The top dial rotates with the integrating wheel indicating the wheel’s precise position at any time. The front dial accumulates slower so can show how many revolutions the integrating wheel has made. The dials are mounted on a frame that moves with the integrating wheel. This can be seen in Figure 6.2, another of Morin’s diagrams. In this diagram, \textit{a\textsubscript{1}} and \textit{a\textsubscript{2}} are the dials.

![Figure 6.2: The wheel mechanism\[dM13, p 56\]](image)

\textsuperscript{1}From the French \textit{roulette}.  
\textsuperscript{2}From the French \textit{tige}.  

\textsuperscript{1}From the French \textit{roulette}.  
\textsuperscript{2}From the French \textit{tige}.  

6.1 Manufacturers

Oppikofer did not construct the planimeter himself but through a Parisian named Ernst [Bro90]. Ernst planimeters are widely reported in the literature and are very true to the designs shown above. What is also interesting are the later copies constructed by Clair, another Parisian “inventor and manufacturer”. To look at, these planimeters clearly follow Oppikofer’s design (see Figure 6.3) but they use a wrapped-wire to convert pen motion to cone rotation.

Figure 6.3: Oppikofer’s planimeter manufactured by Clair [Anoa]

No cone based planimeter was exhibited at the Great Exhibition. However, Clair did exhibit a dynamometer (see Section 1.1.2). This illustrates that Clair was aware of the need to find the horsepower of an engine. It would be interesting to know whether he used his planimeter for calculating the area of indicator diagrams themselves.

The use of a wire rather than a rack-and-pinion mechanism was noted by the jurors

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3Pierre Clair. Inventor and Manufacturer. 93 Rue du Cherche-Midi, Paris. [GE52]
of the Great Exhibition as a location-specific design preference.

*In the Tuscan instrument the motion is conveyed through a rack and pinion, and in those of Swiss and German construction, through a hand and pulley.*

6.2 Modelling an Oppikofer planimeter

Modelling involves making some abstraction of the real world and so the model (described in Chapter 17) is not exactly like the planimeters of Ernst and Clair. The main reason for this is to minimise the complexity of a computer model. The model was constructed using Sasami [Car00] (see Section 14.1.1) and requires a fair amount of processor power. The addition of ‘unnecessary’ features such as the output dials and the track wheels would have significantly reduced the model’s speed.

When the modelling process began Figure 6.3 was not available. Only having the two-dimensional representations of Figures 6.1 and 6.2 resulted in the positioning of the pen’s shaft being on the wrong side of the cone. This has not been corrected as yet, but probably should sometime in the future to improve the mapping between the historical devices and the model. The key components are modelled and contains a metaphorical carriage that conveys the idea of sideways movement. The pen’s position is directly linked to the position of the mouse in an input window and movement of the pen causes the wheel to change position and the cone to roll. The dials are replaced with a revolution counter in a separate ‘output’ window. Despite the necessary abstraction, I think that the model is still identifiable as an Oppikofer planimeter and certainly helps illustrate the diagrams provided in this chapter.
Chapter 7

Kaspar Wetli

Wetli’s planimeter is the archetypal wheel-and-disc planimeter. It was also exhibited at the Great Exhibition and was able to trace areas with superb accuracy. There is a Wetli planimeter in the Science Museum\footnote{Throughout this document “the Science Museum” refers to the Science Museum in Kensington, London.} and pictures can be found in various books and websites. Bromley mentions Wetli’s planimeter developing out of the need to integrate negative functions [Bro90].

Wetli was an engineer working in Zürich in 1849 when he decided to try to develop a planimeter that could handle negative values. His first approach was “using two cones with their vertices opposite so that the upper edges of both formed one straight line” [Hen94]. Such an arrangement allows for the wheel’s position to move beyond the zero-displacement point and on to the opposite cone where accumulation of the wheel would be in the opposite direction (and hence negative).

In such a set-up, sideways motion would cause one of the cones to rotate in one direction and – through a gearing mechanism – the other cone to rotate in the opposite direction. To overcome the difficulty of providing a smooth surface over which the integrating wheel could slip, Wetli placed a disc on top of these cones. The integrating wheel then rolled over the surface of this disc which in turn was driven by the action of the cones under it. The next stage in the development was to eliminate the need for cones
altogether by using the movement of the tracing point to directly cause rotation of the wheel. Wetli chose to accomplish this mechanism by using a wire wrapped around the spindle of the disc much like that of Clair’s (see Section 6.1).

Wetli’s design moves the disc underneath a stationary integrating wheel and this creates the variable gear necessary for mechanical integration. The disc is on a carriage mounted on three tracks and connected to the tracing point. Motion of the tracing point in one direction causes the carriage to move (and therefore change the gear ratio of the integrating wheel) and motion in the other direction causes the disc to spin.

7.1 Wetli at the Great Exhibition

The Welti planimeter exhibited in Crystal Palace in 1851 was awarded a prize medal. This planimeter was manufactured and exhibited by James Goldschmid, a manufacturer based in Zurich [GE51, p1272].

The jury report provides us with some information about construction and accuracy:

“The disc is of glass, covered with paper, and receives the movement of rotation by suitable and simple mechanism. The results obtained by this machine have been found to be correct within 1-1000th part of the area.” [GE52, p304]

---

2Those attentive to the mechanism that causes the disc or cone to rotate will notice that Wetli’s method is identical in principle to Hermann’s. In such mechanisms, the arm of the tracing pen causes the spindle of the wheel (or cone) to rotate. The rotation can be caused by either a wrapped wire (Wetli) or a rack and pinion (Hermann).

3“...for calculating mechanically the area of planes, whatever may be their figure” [GE51, p1272].
7.2. THE WETLI-STARKE PLANIMETER

This accuracy is astonishingly high – even unexpected – when one considers the use of paper as a measuring surface.

7.2 The Wetli-Starke planimeter

Wetli’s design of a wheel-and-disc planimeter was manufactured by Georg Christoph Starke in Vienna [Bro90] and presumably a sizable number of these were made.

The Science Museum have a Wetli-Starke planimeter that was “constructed about 1860” [Anob]. A photo of this planimeter is published in Bromley’s article [Bro90]. In this design the original need for covering the disc with paper has been overcome by using a disc “with a specially prepared fine upper surface” [Anob].
7.3 Improvements at Seeberg

The Wetli-Starke planimeters were improved by Hansen, an astronomer of the Seeberg observatory near Gotha in Bavaria [Hen94]. This new design – manufactured by a local manufacturer named Ausfeld – used a magnifying tracing lens rather than a tracing point and were “instruments of very great accuracy” [Hen94]. One of Ausfeld’s planimeters was exhibited at the Great Exhibition.

7.4 Modelling a Wetli planimeter

A number of models using the wheel and disc principle have been made (See Chapters 15, 16 and 19) with varying complexity.

Of these models, the one illustrated in Figure 7.3 is historically the most accurate portrayal of an actual Wetli planimeter and is documented in Chapter 19. To aid clarity certain elements like the wire that causes the disc to rotate have not been represented.

![Figure 7.3: 3D model of a Wetli planimeter](image)

7.5 Richard brothers’ planimeter

After discussing the Wetli planimeter, Morin [dM13] mentions a planimeter manufactured by the Richard brothers in Paris. The planimeter is designed to find the area of functions
drawn on cylindrical card. It is unclear from where it inherited its wheel and disc mecha-
nism but it is certainly possible that it may have been from various manufactured forms
of Wetli’s planimeter.

Figure 7.4: The Richard brothers’ planimeter [dM13, page 61]

This planimeter is particularly interesting since it employs two discs in the wheel-and-
disc mechanism. Presumably the second disc has the function of forcing the wheel to roll
with improved contact friction.

Figure 7.5: Using a wheel with two discs [dM13, page 62]
Chapter 8

John Sang

For the next chapter in the planimeter’s history our attention is drawn to the Scottish town of Kirkcaldy and Christmas Eve 1851. There we find a Mr John Sang putting the final touches to an academic paper entitled “Description of a Platometer, an instrument for measuring the Areas of Plane Figures.”

This paper was presented to the Royal Scottish Society of Arts (RSSA) in the following January. The platometer is what more generally would be called a rolling planimeter (see Section 3.2) and consisted of a cone mounted on a rolling carriage. It has been claimed that the earliest form of rolling planimeter was constructed by Coradi of Zurich [CU14, p200] however Sang’s platometer is clearly in the same class of mechanism and was developed some 30 years earlier.¹

Sang had exhibited the first of his devices at the Great Exhibition of 1851 under class X (philosophical instruments) and James Clerk Maxwell later recalled that seeing the platometer in Crystal Palace “greatly excited my admiration” [Max55b]. Since exhibiting, Sang had made further developments and in his paper he refers to five models of platometer. The report from a committee of the RSSA in response to Sang’s paper was also very complimentary of his “most ingenious and beautiful instrument” [RSS52].

¹Carse and Urquart’s article on planimeters attempts to harmonise the different developments that went on during the 19th century and both Sang and Coradi are referred to. However, in the article is published a picture of instruments supplied by Coradi [page 200] and it is the caption of this figure that claims it to be of the first rolling planimeter. What it should really say is that it was the first Coradi rolling planimeter.
The question remains as to where Sang’s inspiration came from. Up until this time, the repeated developments of the planimeter were all happening in mainland Europe. Sang’s work is central to the later input of Maxwell who in turn was the motivating inspiration for Thomson’s integrator. However, this spark of activity in the Scottish lowlands is often missed out of the standard histories. From a French prospective, Morin comments on the work of Thomson but is unaware\(^2\) of the work of either Sang or Maxwell [dM13]. Carse and Urquart mention Sang but only with regard to Sang being the trigger of Maxwell’s work [CU14].

It is interesting that both Maxwell and the RSSA committee are under the opinion that the platometer was new research. The RSSA talk of Sang’s “inventive genius” and it is unlikely that they would deliberately ignore previous work in the field that they were aware of.

### 8.1 Sang, an isolated development?

It seems difficult to believe that Sang could have exhibited at the Great Exhibition and not come across the other planimeter exhibitors. It is true that Sang did not use the title

\(^2\)It is always possible that Morin might have been aware of Sang or Maxwell’s work but just didn’t include it. However, *Les Appareils D’Intégration* is in most respects a very complete text including some planimeters that are mentioned nowhere else. It is doubtful that Morin would have excluded either Sang or Maxwell.
8.1. SANG, AN ISOLATED DEVELOPMENT?

planimeter. In fact, in Crystal Palace, he did not even exhibit with the title platometer; preferring the slightly more intuitive name planometer [GE51]. However, even though Sang did not adopt a standard name, the jurors of class X (Philosophical Instruments) grouped the area-measuring devices together in a report entitled Planimeters [GE52]. Sang’s planometer is the first to appear in this report. To remain ignorant of the other instruments at the Great Exhibition, Sang must have never seen the jury’s report of his device. One can hardly imagine that Sang would not have read these reports but evidence would appear to disagree. In the report appended to his paper, a committee of the RSSA express the following interest in the other planimeters that were exhibited.

"Since the subject was last before the Society, a wish has been expressed that your committee should endeavour to assertain the nature of the Tuscan and French instruments, exhibited in the Great Exhibition... there appeared to be an impression on the minds of the members that there was an identity of principle between it and the instruments in use in France."

[RSS52]

The committee wrote to Sang to ask if he had any details. He did not, and appeared to never have looked in to how these continental devices worked.

"The juryman who was appointed to try the working of these machines told me that besides those of Professor Gonella’s construction there was no other, and that there were none on the same principle as mine... A very intelligent gentleman, who had the charge of one of Gonella’s, assured me that there was nothing of the same principle as mine, but that there was a dynamometer acting on somewhat the same principles, which I saw."

[RSS52]

The committee concluded that “even should it turn out that the French instrument was similar in principle... [we] have a thorough conviction that, as far as Mr Sang is concerned, the merit of originality would no less belong to him.” [RSS52]

It is fairly clear therefore that Sang is correctly placed in the list of inventors (see Section 2.1.1). His work was independent, and original. It was not until the planimeters
of Coradi and Ott that anyone else conceived of a rolling planimeter that would not require a track and was not therefore limited to a certain length of displacement.

8.2 Sang at the Great Exhibition

Sang’s contribution to the catalogue of the Great Exhibition is comparatively large and includes a picture. It is described as a “planometer, or self-acting calculator of surfaces” [GE51]. In the introduction, the commentary gives some indication of the uses of a “planometer”. The early planimeters were far more concerned with the problem of calculating area rather than the evaluation of integrals and the catalogue talks of the device being suited to “…the use of surveyors of land and engineer, and also calculated to assist students of physical geography, of geology, and of statistics.” Applications of this type are very much to do with area calculation, although the reference to statistics could be viewed as the integration of distribution curves. However, even then the planometer is not seen as a mechanisation of the calculus; statistical distributions are generally very difficult to integrate and are usually always tabulated using numerical methods. The planometer is more seen as a tool that allows areas to be calculated “…from the best maps, with little trouble” [GE51].

Sang’s device is a cone-based rolling planimeter (see Chapter 8). A small pointer is used to trace a shape and the output is read off a silver index wheel. This index can be reset before tracing but it is recommended that subtracting the end value from the start value is more preferable. The scale on the index wheels can display an area between zero and 100 square inches to a precision of hundred parts. Because of this scale and its width, this planimeter can only measure figures 4.5 inches wide and 22 inches long before an overflow situation occurs. In the jury report, it was noted that errors could be corrected by calculating the area a second time with the instrument reversed and taking an average. This has the effect of averaging out the errors introduced due to the asymmetry of the cone producing a result which “…will be very nearly the truth.” [GE52]

Unlike his European counterparts, Sang’s instrument does not rest on a track but instead relies on the rolling of two heavy wheels over the paper. This did not compromise
accuracy, all the planimeters exhibited appeared “...free from any tendency to divergence in this respect.” [GE52]
Chapter 9

Maxwell

Maxwell chooses to call his device a *platometer* and thus firmly asserts his design to be a development of Sang’s. We are lucky to have available both his published paper and a draft of the same work. In the published paper Maxwell refers to Gonnella’s¹ “large work on *platometers*” [Max55a, emphasis added]. It is particularly interesting that he doesn’t talk about a “large work on *planimeters*” – clearly he is trying to encourage the use of his and Sang’s title of the instrument. This is even more strange when one considers how the word *planimeter* conveys so much more meaning about the application.

Maxwell is clearly writing with an RSSA bias, he chooses to publish the description of his platometer with them and Sang gets used as the principle named example throughout. This is perhaps because he expected Sang to be the main source of identifiable planimeter knowledge in the RSSA.

> “As many members of the Society will have seen Mr Sang’s Platometer and heard him explain it they will be the better prepared to follow what I have to say about these instruments in general.”²

[Max55b]

Interestingly, although Sang’s Platometer is the only other device to be referred to

¹He spells this name *Gonnellu*.
²This sentence is omitted from the published paper.
by name\textsuperscript{3}, he uses a description of a wheel-and-disc integrator to illustrate the principle and adds the passing comment that in Sang’s “The first disc is replaced by a cone” \cite{Max55a} indicating that Maxwell considered Sang’s to be a member of a wider class of instrument. It is also interesting that the wheel-and-disc integrator that he describes remains unattributed to a single inventor despite the fact that an illustration of it appears.

9.1 Motivations

While for Sang, the achievement was producing a working solution that had a practical application, Maxwell was far more concerned with the theory of such a machine. Maxwell’s platometer, which was never constructed, was to him far more an object of academic interest rather than practical importance. The motivation for design was to eliminate the errors introduced into planimeters like Sang’s due to having components that slip.

In a letter to Cecil James Monro dated 7 February 1855 \cite{Max55c} we learn that his paper was written while supervising his Father’s medical treatment. Missing Cambridge and his mathematics, Maxwell’s platometer goes some way to filling a void.

\textit{“I may be up in time to keep the term and so work off a streak of mathematics wh. I begin to yearn after. At present I confine myself to Luck Nightingale’s line of business\textsuperscript{4} except that I have been writing descriptions of Platometers for measuring plane figures and privately by letter confuting rash mechanics who intrude into things they have not got up and suppose that their devices will act when they cant.”}

\cite{Max55c}

A look at the diagrams published in his paper \cite{Max55a} shows the elaborate construction that Maxwell had in mind. It is quite easy to imagine “rash mechanics” not

\textsuperscript{3}This is especially interesting if one considers how little Maxwell’s device corresponds to Sang’s. With the exception of the fact that it too is presented as a rolling planimeter (with the exception Figure 4), Maxwell’s platometer is conceptually closer to a wheel-and-disc integrator. For example, Maxwell’s platometer could perform negative integration.

\textsuperscript{4}A reference to Florence Nightingale.

\textsuperscript{5}Spelling preserved as in \cite{Max55c}.
9.2. **CONSTRUCTION**

It is apparent that Maxwell did have every intention of constructing his platometer. He doesn’t just offer the principle of the integrating mechanism but actually provides us with two designs for implementation.

After the presentation of Maxwell’s paper, a committee of the RSSA recommended [Har90] that the society should award expenses of no more than £20 towards the construction of the platometer. The actual award was £10. In a letter to his Father, Maxwell has a clear intention to construct the platometer.

“I got a note from the Society of Arts about the platometer, awarding thanks, and offering to defray the expenses to the extent of £10, on the machine being produced in a working order. When I have arranged it in my head, I intend to write to James
Aware of the complexity, his Father was doubtful that the platometer could be made to budget.

“The platometer will require much consideration, both by you and by anyone that undertakes the making. You need hardly expect the details all rightly planned at the first; many defects will occur, and new devices contrived to conquer unforeseen difficulties in the execution. I would suspect £10 would not go far to get it into anything like good working order. If the instrument were made, to whom is it to belong? And if it succeeds well, for whose profit is all to be contrived? Does Bryson so understand it as to be able to make it? Could he estimate the cost, or would he contract to get an instrument up? Fixing on a suitable size is very important.”

[CG82, pp114-5]

Presumably, the cost of manufacture did exceed £10 and Maxwell’s concept never became physical.

### 9.3 An alternative version

Maxwell’s first device was a rolling planimeter and strongly resembled Sang’s principle; but in his paper he introduced the design for a second use of his mechanism. Based on quite a different principle, this second platometer had a tracing arm that caused a combination of hemispherical rotation and spherical rolling. As this arm is rotated about the centre axle, the hemisphere rotates and this movement then causes the sphere to move. The position of the sphere on the hemisphere changes the gear ratio of this movement and maps to inward-outward movement of the tracing point. Henrici classified this as a Class II planimeter (a planimeter that uses polar co-ordinates but is not of the Amsler-type) [Hen94].
Figure 9.2: Maxwell’s (class II) planimeter [Max55a]
Chapter 10

Thomson and Kelvin

James Thomson was the brother of Lord Kelvin, the famed nineteenth century physicist. Thomson conceived of his disc-globe-cylinder integrator “some time between the years 1861 and 1864” [Tho76a]. Thomson wanted to incorporate Maxwell’s concept of using pure rolling into a device that “might be simpler than the instrument of Prof. Maxwell and preferable to it in mechanism” [Tho76a].

Much like Maxwell, Thomson was very interested in the design of the instrument; although he had the confidence that it would “prove valuable when occasion for its employment might be found” [Tho76a]. Well over a decade passed before his brother would supply such an application.

Lord Kelvin was a prominent scientist and inventor of his time. He encouraged the use of model construction in the understanding of science.

“I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get the electromagnetic theory ..... But I want to understand light as well as I can, without introducing things that we understand even less of. That is why I take plain dynamics. I can get a model in plain dynamics; I cannot in electromagnetics.”

[www04b]
In 1876, Lord Kelvin reflected on the need to mechanise Fourier analysis on a given function and while discussing his ideas with his brother, it came to light that the means were available.

“During many years previously it had appeared to me that the object ought to be accomplished by some simple mechanical means.”

“[H]e described to me in return a kind of mechanical integrator which had occurred to him many years ago, but of which he had never published any description. I instantly saw that it gave me a much simpler means of attaining my special object than anything I had been able to think of previously.”

[Tho76b]

Figure 10.1: Thomson’s disc-globe-cylinder integrator [Tho76a]
Kelvin then moved with rapid speed. Within days of this inspiration, four influential papers were prepared for presentation to the Royal Society of London. The first was published by Thomson and described his integrator in detail, the remainder were by Lord Kelvin and discussed their uses. These papers testify to the significance of Thomson’s integrator. Suddenly, Lord Kelvin could integrate products [Tho76b] and solve second order differential equations [Tho76c]. In fact, with a particular set-up, he could solve differential equations of an arbitrary order [Tho76d]. The last paper concludes with a powerful remark about the significance of this mechanisation.

“Thus we have a complete mechanical integration of the problem of finding the free motions of any number of mutually influencing particles, not restricted by any of the approximate suppositions which the analytical treatment of the lunar and planetary theories requires.”

[Tho76d]

The development tradition of Sang, Maxwell and the Thomson brothers made a great impact on the British history of planimeters. We should be thankful that Kelvin published his work with the Royal Society, otherwise the history of pure-rolling planimeters would never have become so well recorded.
Chapter 11

From Planimeter to Differential Analyser

Although Kelvin conceived of how mechanical integrators could be connected together to solve differential equations, a device that employed the principle was not available until Vannevar Bush invented the differential analyser in 1927.

Kelvin’s design required integrating units to be able to drive other integrating units. In general, the output shaft of a planimeter does not have the necessary torque to act as an input for another. What was required was a torque amplifier to provide the interconnections with the extra turning force required.

The torque amplifier was developed by Niemann but is effectively the mechanical principle behind the technology used on ship capstan wheels [Bro90]. Various means of torque amplification were used in the differential analysers. In later years the Niemann amplifier was replaced by ‘follow up’ systems where a servo-motor ‘followed up’ the integrating wheel [Fif61b].

11.1 Integrators used in Bush’s differential analysers

The integrating wheels used in Bush’s differential analyser were of the wheel-and-disc variety. Much like Wetli’s planimeter, these integrators consisted of a disc that moved
under a stationary wheel. The position of the disc was controlled by a large positioning screw that was the single most expensive component [Cra47].

### 11.2 The Manchester differential analyser

Manchester University had a differential analyser installed in 1935. The machine was based on Bush’s design and consisted of eight integrating machines linked to a number of input and output tables [Cra47]. Figure 11.1 shows the differential analyser with a close-up shot of one of the integrator units. The mechanism marked H is the integrator’s torque amplifier. This machine is now on display at the Science Museum.¹

¹Part of the Manchester Differential Analyser was retained by Manchester University. A picture of the Science Museum exhibit can be found on-line [www00a].
Chapter 12

Amsler

The Amsler, or polar planimeter, is not the focus of this work since it is not part of the planimeter history linked to the differential analyser. However, as the project developed it became increasingly evident that some treatment of the history of these devices could not be avoided. It is the purpose of this chapter to break away from the main motivations of the project and to look at a device that became – in the engineering world – as important as the slide-rule.¹ Many manufacturers were very successful in producing polar planimeters and some still exist today. Perhaps the most notable thing about Amsler is his success as a manufacturer himself. Between its invention in 1854 and 1892 Amsler had constructed and sold over 12,000 polar planimeters from his factory in Schaffhausen, Switzerland [Hen94].

12.1 The Amsler polar planimeter

Amsler’s planimeter design remained fairly unaltered throughout the nineteenth century although he did design and construct more complicated planimeters such as his “moment planimeter” [Hen94]. The polar planimeter consists of two arms connected together at a

¹The slide rule is a calculating instrument based on the principles of logarithms. In my copy of *Practical Mathematics* [Cas51] the chapters relating to the operation of a slide-rule constitute seven pages of text. However, the measurement of irregular areas (which include a number of pages on the use of planimeters to accomplish this task) is nearly twenty.
pivot. One end of the construction is fixed to stationary point and the other end provides the tracing point.

![Amsler’s polar planimeter](Cas51)

As the angle between the two arms change, the angle between the wheel and the radial path of the hinge with respect to the anchor point changes. This constitutes the variable gear essential for integration.

### 12.2 Modelling the polar planimeter

Although the polar planimeter is not the emphasis of the project, a simple model has been produced. It is two-dimensional and documented in Chapter 20.

### 12.3 Developments of the polar planimeter

#### 12.3.1 Coradi

The polar planimeter was developed by a number of manufacturers who improved either the accuracy or usability. Coradi, a Zurich based manufacturer, incorporated much innovation into his planimeters. His ‘precision planimeter’ positioned the integrating wheel on a “finely prepared surface” [Hen94] to eliminate errors caused by variations in the tracing surface. Coradi also developed the idea of the ‘rolling planimeter’.\(^2\)

\(^2\)This principle was first invented by Sang. Coradi’s rolling planimeters are polar whereas Sang’s are orthogonal (see Section 3.2).
A second major improvement on Amsler’s model was the ‘compensating polar planimeter’, a planimeter with a detachable arm. By averaging area calculations made with the arm in different positions, a more accurate value could be obtained [CU14].

Figure 12.2: A compensating polar planimeter tracing an indicator diagram

12.3.2 Willis

Earlier it was noted that a principle use of the planimeter was finding the area of indicator diagrams. Another development of the Amsler principle was the planimeter developed by American inventor Edward Jones Willis. Willis’ planimeter had a numerical scale that allowed the user to “read the mean effective pressure (MEP) or horsepower without calculation of any kind and without application of measurements to the diagram” [Sch93].
Part II

Illustrating the History
Chapter 13

Computer Models as History

A large portion of the work undertaken in this project has been to do with using software to illustrate the history of a historical device. This was accomplished using tools associated with the modelling process that is known at the University of Warwick as ‘empirical modelling’. One of the things that has been in the background of this project from the very beginning is how a computer can help people to understand devices like the planimeter. During a visit to the Science Museum, I was able to experiment with some real-life planimeters and was astonished how much it aided my understanding of their workings. What is required is something to bridge the gap between diagram and reality. Here I believe is the developing role of software in historical study. By providing the historian with the means to interactively illustrate their subject, the reader is able to experience something close to the real thing.

To illustrate how the planimeter could be modelled, a number of models have been produced. They cover the Wetli and Oppikofer style planimeters. The various models have experimented with the different levels of abstraction that can be used to take the planimeter to illustration. For example, some models are have the emphasis of looking and feeling like planimeters, whereas others more simply focus on the mechanical principle at work rather than the device as a whole.
13.1 Existing models

There are a number of websites\(^1\) that provide information about the history and uses of planimeters. One of these contains an applet illustrating how the polar planimeter calculates area. This model is geared towards understanding the principle rather than visualising the actual historical devices. Although such a technique was important, I decided that the main focus of the models should be towards bridging the gap between concept and device, easing users in their visualisation of historical objects.

\(^1\)www.google.com returns around 13,000 sites related to the keyword ‘planimeter’. All the sources found on the web are referenced on Google.
13.2 The modelling tools

The empirical modelling tools used are interpreters for the Eden language. Some models use TkEden, and others dTkeden (dTkEden is a distributed version of TkEden).

13.3 The modelling process

The process of modelling a real-world system on a computer requires some form of abstraction of the world. Classically, this abstraction has been undertaken by developing a mathematical model of our understanding of the problem. Such a model is often developed on paper, forcing considerations about complexity and flexibility to become fixed before any useful code is written. The approach of empirical modelling is to take a real-world system as it exists and to start modelling its interactions based on what is observable. In essence, empirical modelling replaces the rigid mathematical interpretation of a system’s inner-workings with a flexible set of definitions relating to the system’s outward-characteristics.
Chapter 14

Programming Languages

At the start of the project, the first few weeks were timetabled to make some decisions about what programming language to use for producing models of the planimeter. The two technologies shortlisted to be considered were Java and Eden. Often the difference between various languages is merely one of preference and it is fair to say that this set of choices were the intersection of my comfort-zone languages and those platforms for which support was available for. One obvious question would be why I settled on any conventional programming language at all. The mechanical nature of my models might have lent themselves to some CAD-style engineering modelling tool, and perhaps this should have been considered. The problem with such a tool would be the viewing and manipulation of the model. It is important that users will be able to interact with the model without the need for rare or expensive software.

14.1 What was needed

From early on in the project it was clear that certain things were important for the different models to be as intended. The language of choice needed to be able to support a number of features.
14.1.1 3D visualisation

To achieve their full potential, the models should have some kind of three-dimensional visualisation. Some models could have been (and were) given a two-dimensional representation but half the excitement of producing models of planimeters was that they would look the part and it was felt that this meant not restricting the output to two dimensions.

This provided two possible choices of 3D renderer and a number of possible languages.

- Java with Java3D
- C++ with OpenGL
- Eden with OpenGL (Sasami notation)

The Sasami notation is the name of the extension to the Eden framework that provides notation for defining and manipulating OpenGL objects.

14.1.2 Dependency maintenance

The reason why Eden was on the list of possible languages was because of one element of its programming philosophy that I felt would help to model a mechanical device intuitively. This interesting feature is what is know as dependency.

Dependency, an example

In a conventional language, variables are given values that persist until an update to that variable is made by the program.

\[
\begin{align*}
A &= 3; \\
B &= A+1; \\
A &= A+1;
\end{align*}
\]

In this example, we define the variable \(A\) to the integer 3 and \(B\) to the value of the sum of \(A\) and the integer 1. After this \(A\) is updated to be “its old value plus 1”. The end result is that \(A\) is equal to \(B\). This is a useful and classical way to program. However, it could be that the programmer is attempting to maintain \(B\) as “\(A\) plus 1”. In this case, Eden’s dependency operator would assist.
C = 3;
D is C+1;
C = C+1;

The keyword *is* provides us with just the necessary semantics. Line 2 in this fragment has the meaning “maintain the variable D so that its value is always the current value of C plus 1”. This has the result that the end values of the fragment are C=4 and D=5.

**Why dependency?**

Dependency is a vital tool in modelling mechanical systems since it allows the connection of components to be modelled. For example, a wheel rolling on a disc moves when the disc moves. This is a dependency, the wheel’s rotation is *caused* by a change in the disc’s rotation.

The other helpful aspect of dependency is that it allows state to be associated in meaningful ways. An example could be the length of a planimeter’s pen; such a dimension seems irrelevant but its magnitude was most likely chosen because of the height of the pen’s shaft above the paper. Dependency can allow the pen length to be defined in terms of the height of this shaft off the paper and so ensure that the pen is always in contact with the paper.

**Dependency in Java**

The ability to use dependency is not limited to Eden, there are Java tools available that enable the use of dependency maintenance. One such example is JaM, a tool that was mentioned in the project’s initial specification.

**14.1.3 Evaluation of languages**

It was the intention at the start of the project that the choice of language for implementing the models should be postponed until some evaluation work had been undertaken. This evaluation would assess how well different paradigms would cope with the needs. This evaluation work never developed as far as intended. Java3D was a very difficult framework
to start using and is very much geared to producing something for which there is a well-defined specification. On the other hand, empirical modelling tools allow the modeller to form ideas while still experimenting with different representations of those ideas. As well as finding the Eden language easier to use, there was also the added benefit that seminars were being provided for third year project students using the tools. The final decision to use Eden was probably less of a decision and more of where I had ended up. I do feel however that some future work (see Section 21.5) could be done in taking my Eden models and producing models that could run without the installation of an environment such as TkEden. A Java program would have provided portability.

14.2 What would have been useful

14.2.1 Portability

Although not necessary for what was produced it would have been nice if the models could have been more portable. Writing the models in TkEden requires the installation of the TkEden environment on a machine before the models can be run. The purpose of this project was to investigate how the models could be developed to aid the history so it isn’t such of a concern that the models are not as portable. In fact, Java3D isn’t part of the standard Java distribution, so even developing the models in Java would have been unsatisfactory. Perhaps the best way forward would be to develop compiled models in something like C++ and to use OpenGL which is available on practically every platform.

The other option would be to provide a viewer version of TkEden. Like Adobe Acrobat Reader\(^1\), this could be a browser plug-in, enabling Eden models to be run on a machine without the need for TkEden (which is a full development environment) to be installed.

Alternatively, a compiler for TkEden models that could produce some executable code might be a useful tool.

\(^1\)Available at http://www.adobe.com/acrobat
14.2. WHAT WOULD HAVE BEEN USEFUL

14.2.2 Support for curves and spheres

Sasami was submitted as a third year project in 2000. In the original plan of that project there was an intention to include what was described as ‘utility functions’ [Car00] which would provide the means to define and manipulate objects richer than basic polyhedra. In the final version of Sasami, just one class of these methods were implemented. These are the two methods that allow a Sasami object to be loaded from an .obj file. This is a standard 3D graphics format and consists of collections of many vertices and faces.

The lack of these utility methods to construct discs was at first an indication that Sasami would not be able to support such structures in a way suitable for building planimeters. (Recall that all the planimeters that were to be modelled all had at least one component as a wheel.)

The solution for including curved shapes is to build the structure in a 3D graphics program and to import it as an .obj file. This technique was illustrated by Carter himself in his model of a billiards table (see Figure 14.1).

![Figure 14.1: Sasami visualisation of a billiards table [Car00]](image)

The close-up screen shot reveals that the spherical balls are really polyhedra. Of course, all computer graphics are reduced to triangles when they are rendered and all an
.obj file provides one with a convenient way of introducing a large number of polygons into Sasami and attaching them to a single object. In the models of planimeters, the imported objects needed a far higher resolution than the sphere objects used in this model.

.obj files were used in all the 3D planimeter models produced. Their creation is discussed in Section 15.1.1.
Chapter 15

Wheel and Disc Model (3D)

The first model that was constructed was more illustrative of the wheel-and-disc principal than of an actual planimeter. It provided visualisation of the wheel-and-disc in 3D using Sasami.

Figure 15.1: The wheel and disc
15.1 Challenges in the construction

15.1.1 The wheel and disc

The wheel and disc are imported .obj files as described in Section 14.2.2. They were created using the engineering design tool ProEngineer.\(^1\) In ProEngineer, a GUI interface is used to create objects on screen and the export process chooses how many triangles to evaluate. Both objects have around 700 triangle definitions to describe their structure and so have a resolution that makes them very effective at representing wheels or discs. The object used for the integrating wheel – marked in this model with a red square (see Figure 15.1) – is slightly different from the bigger disc in that it has an edge that tapers off to a point.

Both disc objects have a radius of 5 units and a width (before any tapering) of 0.2 units when they are imported.

Figure 15.2: The disc representing the integrating wheel is tapered

15.1.2 Temporal dependency

It has already been claimed that the dependency tools available in the Eden language are a useful aid to modelling mechanical systems. In these systems, different components of

\(^1\)I would like to draw attention to the help provided by Chris Burland (School of Engineering) and to thank him for helping me create the .obj files used in my models.
a mechanism are connected to others by some kind of power transmission like cogs and shafts. These connections represent a mechanical dependency that can be mapped into an Eden dependency.

In an initial attempt at this model, such an approach was taken. Mouse motion in the input frame was linked through dependency to the displacement of the integrating wheel and the rotation of the large disc.

\[
\text{wheelRatio is drawPanel\_mousePos[1];}
\]
\[
\text{discRotation is drawPanel\_mousePos[2];}
\]

It then makes sense to define the rotation of the wheel in terms of these. The wheel rolls over the disc and the amount it rolls depends on the displacement of the integrating wheel from the centre of the disc. When the wheel is at the centre of the disc (\(\text{wheelRatio} = 0\)) the wheel remains stationary and when the wheel is at the edge of the disc (\(\text{wheelRatio} = 1\)), it experiences maximum rotation.

\[
\text{wheelRotation is discRotation*wheelRatio;}
\]

However, on closer inspection this is not correct at all. The variable \(\text{wheelRatio}\) provides us with a value that indicates the gear ratio between the disc and the wheel at any point. What the above dependency does not take into account is the fact that this variable can (and most certainly will) vary throughout the runtime of the model. In this example \(\text{wheelRotation}\) will be updated each time the wheel is moved and in English is “the amount the wheel has rotated since the beginning of the planimeter’s runtime, given that the wheel was positioned in the position corresponding to \(\text{wheelRatio}\)”.

In order to deliver the required semantics of this variable – where it shows the cumulative rotation over time – a construction was required that could allow the accumulation of values given by a dependency that requires knowledge of its history.

This code is known as the clock or accumulator procedure in the various models and is common to all the different types of planimeter.
15.2 Temporal variables

In order to model this relationship it is necessary to represent the varying quantities as quantities that vary with time.

When integration is introduced to a class of mathematics students; it is common to talk of changes happening in a small interval \((\delta x)\), which is then minimised until it becomes infinitely narrow \((dx)\).

\[
\int f(x)dx
\]

This is represented on the planimeter when \(x\) is the movement of the pen along the track and \(f(x)\) is the movement perpendicular to the track.

If the translational movement along the track is seen as a function of time, \(x = \chi(t)\) then for a given time interval \([t_0, t_0 + \delta t]\) the area under the curve is given by:

\[
(\chi(t_0 + \delta t) - \chi(t_0))(f(\chi(t_0))
\]

and as this interval is minimised:

\[
\int f(x)dx = \int f(\chi(t))\chi(t)dt
\]

The generalisation allows both \(x\) and \(f(x)\) to be represented by computer variables that are adjusted after some small amount of time \(\delta t\).

The clock procedure for a model of a cone planimeter is given in Section 17.3.

15.3 How to run this model

1. Load the script into TkEden

\[$ /dcs/emp/empublic/bin/tkeden model1.eden \$

2. Initialise the mouse by moving it into the input frame.
3. Initialise the time counter by executing the `startClock` Eden procedure. This starts the sampling process and allows the mechanism to start working.

```
startClock();
```

4. Mouse movements in the input window (the square with axis painted on it) should now map to the movement of the planimeter’s tracing point.

While tracing, the movement of the disc and wheel are also illustrated in two dimensions on the control panel. The circle with the blue marker illustrates the position of the disc while the circle with the red marker gives the position of the wheel. The numerical output inside the latter wheel view gives a numerical output corresponding to the area traced.

15.4 Issues relating to this model

This model was presented informally to all the other students who were making use of the empirical modelling tools in their third year projects. In this presentation, it became clear that there were certain issues in the user interface of the model that needed to be addressed.

15.4.1 Reseting the model

In the planimeter models, there is a clear need for the user to be able to reset the area counter or the integrating wheel. This enables a new area to be traced and illustrated on the counter without having to subtract the start reading. This common task should be provided with an associated button. Having to do the reset manually by typing in some code is unacceptable.
15.4.2 Sasami zoom and rotate controls

Because the mouse is used in the input frame, it is necessary to set up the 3D visualisation of the model before tracing an object. Changing the angle of view or zooming (this is done in the Sasami window using the different mouse buttons) requires the mouse to have left the input pane.

15.4.3 Modelling principle or device?

The students who were shown this model found it very difficult to see the wheel and disc as an area calculator. After some minutes of interaction it was possible to convince them that the device worked and it was also possible to see the mechanism as a variable gear. However, the mapping between actions in the input frame and the activity in the Sasami visualisation proved difficult.

I think this was due to two reasons:

1. It was difficult to see the rotation just using the 3D representation.

2. There was no movement of a ‘pen’ in the simulation and so no clear reason why the disc was spinning. The mechanism was just floating in mid air without any inputs or outputs.

For the user acquainted with planimeters, this model is a helpful simulation of the principle of the variable gear as an integrating mechanism. In this sense, the model could be seen as a dynamic version of Figure 3.1. What would have suited the audience unaware of the various planimeter designs would have been to construct the model with more real-world mapping.
Chapter 16

Wheel and Disc Model (2D)

Earlier it was said that the motivation for the project was that planimeters were a central component of the differential analyser in which there are a number of planimeters that can be connected in a number of ways. This model was the start of a more distributed way of looking at the planimeter models where more than one planimeter could be connected to an input table.

A ‘problem’ with the basic wheel-and-disc model was that the manipulating of complex Sasami objects (like discs) in real time is very processor intensive. This model explores how a two-dimensional (and so less processor intensive) version could be constructed.

16.1 Improvements with respect to the first model

This model addresses some of the issues relating to the 3D version that were raised in Section 15.4. It implements a distributed model in which the planimeter is on a separate machine to the input window. In this way one mouse is made available for tracing and another for controlling the planimeter model.
CHAPTER 16. WHEEL AND DISC MODEL (2D)

16.2 Starting the model

The model consists of 3 dTkEden\(^1\) sessions: the server, planimeter and input table. To allow one mouse to be dedicated to the tracing and another to the controls the two clients should be run on separate machines. The server handles communication between the clients and can be run on either (or another) machine. If having a dedicated mouse isn’t important then all the clients can be run on the same machine.

Assuming dTkEden is installed at /dcs/emp/emppublic/linux-i686/bin/dtkeden the model is provided with some bash shell scripts to set up the dTkEden sessions.

1. Start the server
   
   $ ./startServer

2. Start the input table

   $ ./startInputTable1

\(^1\)dTkEden is a networked version of TkEden and allows components of a model to be distributed across a network. It is described more fully in Chapter 18.
3. Sign in to this window with the agent name inputTable1

4. Start the planimeter (may be on a separate machine)
   
   $ ./startPlanimeter1

5. Sign in to this window with the agent name planimeter1

6. Start the planimeter’s clock function by pressing the start button in the clock section of the control panel.

16.3 Using the model

The model provides 2D viewing of the planimeter as a standard engineering drawing. The set of axes to the side of the front elevation indicate, with colours, which view is which. The integrating wheel will spin in proportion to the area being traced if the clock is running. This clock can be left running, however provision for stopping it has been provided so that processor power used by dTkEden can be reduced.

Mouse movements in the input table manipulate the position of the wheel on the disc and the rotation of the disc.

16.3.1 The revolution counter

Initially, the revolution counters show how many revolutions the wheel and disc have made during the run-time of the model. Rotations of these are also illustrated in the above and side views of the model. The reset button resets these counters, yet it doesn’t reset the position of the wheel. This effectively means that the counter shows the number of rotations since the last press of the reset button. The wheel is not reset to a stationary position as such an interaction would break the dependency that the wheel only moves by rolling with the disc. To have such behaviour would be as if the functionality of pressing the reset button on a car mileage counter were to spin the wheels and return them to the positions that they occupied when the car was new.
16.4 Evaluation of the model

This model was demonstrated to Ashley Ward and Steve Russ in December. For Ashley, the model was his first contact with planimeters and it was pleasing how successful the model was at illustrating the principle.

In this model, instead of the integrating wheel sliding over the disc, the disc moves under a stationary wheel. This brought it closer in terms of historical accuracy to Wetli’s design (see Chapter 7).

The reset button is a useful addition since it allows users to trace areas with a counter starting at zero. Providing revolution counters for both the wheel and the disc was more misleading than helpful since it wasn’t clear which one was representing the area traced. It was realised that what was really lacking was a counter that gave the area traced in units.

The double-workstation set up with the input table is on one machine and the model on another worked quite well although it was quite difficult for one person to maintain awareness in both directions. An improved interface might be one where the input table machine is replaced with an interface device like a graphics tablet. The mouse is a difficult tool to draw with. The set-up really comes into its own when the system is operating in a ‘classroom style’ – where an audience is watching a single user interact with the model. Such a configuration allows the audience to see clearly that the device is performing some area calculation and they can watch the effect of different interactions more clearly than if they were the user. This situation is improved further if the audience are encouraged to give tasks to the operator, or if the task of operating is rotated.

The distributed structure of building a model is very flexible and allows for code re-use. Future modifications like swapping the software input table for a hardware graphics tablet could be done quite easily.
Chapter 17

Modelling an Oppikofer Planimeter

It seemed logical that the next model to be constructed should be a cone-and-wheel planimeter. Because this model contains both a cone and a wheel it seemed appropriate to use it to describe the modelling process in more detail.

17.1 A starting point

If such an emphasis is to be put on observation, then a logical starting point is to use a diagram (See Figure 17.1) of what is to be modelled. It is clear from this diagram that this planimeter has a number of distinct components. The process of modelling will involve representing each of these as individual components (Section 17.2), assembling them together, and then linking their interactions (Section 17.3).

17.2 The basic components

17.2.1 The cone

The obj file to be imported for the cone is cone_point.obj and contains definitions of something in the region of 700 triangles.

```bash
%sasami
load_obj cone "cone_point.obj"
```
This imports the polygon and places it at the origin. It is necessary for the uppermost edge of this cone to be horizontal (see Figure 17.1) so the cone is rotated through $15^\circ$. Its position is also linked through dependency to the variable \texttt{carriagePos} which will control the position of the cone as it moves along its track from side to side.

```sasami
object_rot cone_1 15 0 coneRotation
object_pos cone_1 carriagePos 0 0
```

The reason for the choice of a $15^\circ$ rotation is because the object file loads an object in the orientation shown in Figure 17.2. The object needs rotating so that it has the same orientation as the cone in Figure 17.1.

The variable \texttt{coneRotation} will be a value controlling the rotation (in degrees) of the cone. Since \texttt{coneRotation} and \texttt{carriagePos} are Eden variables, they need to be instantiated before the above definitions will take effect.

```eden
coneRotation = 0;
carriagePos = 0;
```

### 17.2.2 The roller

Next it is necessary to add the roller (marked \textbf{P} in Figure 17.1) and to connect it to the wheel by a shaft. The wheel is another imported object \texttt{disc.obj}
17.2. THE BASIC COMPONENTS

17.2.3 The shaft

There will be a shaft connecting the cone and the wheel. Using dependency, it makes sense to define the position of the roller in terms of the position of the cone and the length of the shaft. Here is quite a complicated transformation problem that classically would require the drawing of a diagram and the application of trigonometry. However, because of the way the Eden variables are used, the model actually becomes our diagram. Since the point at which the cone is to be connected to the shaft is the origin; we can look at the problem and start to write down things we know about the distances between points.

```
%eden
```

```sasami
load_obj roller "disc.obj"

This is also set at 15° and its size adjusted to make the desired shape. The size of this roller is scaled by an Eden variable `roller_size` and its position defined by three other variables `roller_pos_x`, `roller_pos_y` and `roller_pos_z`.

```
%python
object_pos roller_1 roller_pos_x roller_pos_y roller_pos_z
object_rot roller_1 15 0 coneRotation
object_scale roller_1 roller_size roller_size 8*roller_size
```
\texttt{shaft\_length=6;}  \\
\texttt{roller\_pos\_x \ is \ carriagePos;}  \\
\texttt{roller\_pos\_y \ is \ -shaft\_length*\sin(PI/12);}  \\
\texttt{roller\_pos\_z \ is \ shaft\_length*\cos(PI/12);}  \\
\texttt{roller\_size = 0.3;}  \\

It should be mentioned here that although Sasami uses degrees for its angle measurements, Eden uses radians in its trigonometric functions. So throughout the code, the equality $15^\circ = \frac{\pi}{12}$ is used.

Now we have the roller and cone. Both objects are linked in their rotation and position by dependency. For example, providing the redefinition \texttt{coneRotation=20;} rotates both by $20^\circ$. The true power of modelling in this way can be seen if the shaft length were defined to be slightly larger. Figure 17.3 shows how redefining \texttt{shaft\_length} from 6 to 10 changes the output.

![Figure 17.3: Re-Defining the Shaft\_length](image)

Next a Sasami visualisation of this shaft is required. Represented as a cuboid, the shaft will be a little longer than \texttt{shaft\_length} so that it gives the impression of going through the roller. Reasons for modelling the shaft as a cuboid rather than a cylinder are twofold; firstly, a cylinder would use something in the order of 700 times the memory, and secondly, rotation of the more angular cuboid shaft is easier to see.

The shaft used is defined with simple dimensions (see Figure 17.4) and is then stretched to the correct size (see Figure 17.5). This has enabled me to use the same Eden definitions
17.2. THE BASIC COMPONENTS

Figure 17.4: Dimensions and visualisation of the unscaled shaft
(with different names) elsewhere in the code for other shafts.

```cpp
%sasami
object shaft

## Vertices at front of the shaft
vertex shaft_a1 -5 5 0
vertex shaft_b1 5 5 0
vertex shaft_c1 5 -5 0
vertex shaft_d1 -5 -5 0
## Vertices at rear of the shaft
vertex shaft_a2 -5 5 0.2
vertex shaft_b2 5 5 0.2
vertex shaft_c2 5 -5 0.2
vertex Shaft_d2 5 -5 0.2

## declaring polygons in terms of vertices
## (notice that the order of vertices define which side
## of each polygon is solid)
poly geom vertex shaft_end1 shaft_a1 shaft_b1 shaft_c1 shaft_d1
poly geom vertex shaft_end2 shaft_d2 shaft_c2 shaft_b2 shaft_a2
poly geom vertex shaft_sideA shaft_a2 shaft_b2 shaft_b1 shaft_a1
poly geom vertex shaft_sideB shaft_b2 shaft_c2 shaft_c1 shaft_b1
poly geom vertex shaft_sideC shaft_c2 shaft_d2 shaft_d1 shaft_c1
poly geom vertex shaft_sideD shaft_d2 shaft_a2 shaft_a1 shaft_d1

## Attach the Polygons to the Shaft Object
object poly shaft shaft_end1 shaft_end2
object poly shaft shaft_sideA shaft_sideB shaft_sideC shaft_sideD
```
CHAPTER 17. MODELLING AN OPPIKOER PLANIMETER

Using this shaft code, the shaft is stretched to the correct size and is placed in the correct position. To model its physical connection with the cone and the roller, it is associated through dependency to the coneRotation variable.

```sasami
% object_pos shaft carriagePos 0 0
object_scale shaft 0.02 0.02 (shaft_length+8*roller_size*0.3)/0.2
object_rot shaft 15 0 coneRotation
```

Figure 17.5: The shaft object after scaling

### 17.2.4 The track

The roller needs to roll on a track. This track is defined in Sasami as an angled wedge.

Firstly the dimensions and positions are declared using Eden variables.

```eden
## Primary dimensions of track
track_width=1;
track_height=0.3;
track_length=20;

## Secondary dimensions of track
track_height_1 is track_height+track_width*sin(PI/12);
track_width_1 is track_width*cos(PI/12);
```
17.2. THE BASIC COMPONENTS

## Track Position

```
track_pos_x = 0;
track_pos_y = 0;
track_pos_z = 0;
```

These dimensions are used to create the eight vertices of the track. Polygons are then created for each side of the track and attached to the track object. Finally, the object is positioned according to the position variables.

```
%asami
object track

vertex tracka1 0 0 track_width_1
vertex trackb1 0 0 0
vertex trackc1 0 track_height_1 0
vertex trackd1 0 track_height track_width_1

vertex tracka2 -track_length 0 track_width_1
vertex trackb2 -track_length 0 0
vertex trackc2 -track_length track_height_1 0
vertex trackd2 -track_length track_height track_width_1

object_pos track track_pos_x track_pos_y track_pos_z
```

17.2.5 The wheel

The integrating wheel (marked R in Figure 17.1) rolls on the uppermost surface of the cone as the cone rotates. Much like the cone, the wheel also has an angular displacement:

```
%eden
wheelRotation = 0;
```

The wheel is an imported object with diameter 10 units.

```
...%asami
load_obj wheel "disc_taper.obj"
object_pos wheel_1 wheel_pos_x wheel_pos_y wheel_pos_z
object_rot wheel_1 0 0 wheelRotation
object_scale wheel_1 wheel_size wheel_size wheel_size
...
CHAPTER 17. MODELLING AN OPPIKOFER PLANIMETER

This wheel is allowed to slide along the surface of the cone as the function being integrated varies.

The wheel’s position is represented by the variable \( \text{wheelPos} \) which ranges over the interval \([0,1]\).

\[
\text{wheelPos} = 0;
\]

If \( \text{wheelPos} = 0 \) then the wheel is at the point of the cone and if \( \text{wheelPos} = 1 \) the wheel is at the base of the cone. The variable that defines the \( z \)-coordinate of the wheel is linked to this variable through dependency.

\[
\text{wheel_pos_z} \text{ is } 2.5 \times \sin(\pi/12) - \text{wheel_width} - 9.85 \times (1 - \text{wheelPos});
\]

In this code the numerical constants refer to the dimensions of the cone.

17.2.6 The pen shaft & carriage

As well as the defined components, the actual model also includes a carriage and a shaft that links the wheel to a pen. These can be seen Figure 17.1, the carriage being component \( A \) and the shaft being component \( T \).

Figure 17.6 shows all these components assembled together.

17.3 Dynamic rolling

So far, the modelling has been static, that is to say the components have been placed in various places around the model. The rotating components have been defined in terms of an angular displacement from some starting position.

In order to get the planimeter to perform integration, the model needs to become more dynamic. It is necessary, for example, to model a small change in \( \text{coneRotation} \) and to see this change as the cause of a proportional change in \( \text{wheelRotation} \).

A variable representing the rotation of the cone due to the roller rolling over the track is given by \( \text{coneSpindleRotation} \). This is directly linked through dependency to the roller’s position.
coneSpindleRotation is \(-360 \times \text{carriagePos}/(2.5 \times 2 \times \pi)\);

The actual rotation displayed on the screen is given by \text{coneRotation} which is synchronised with \text{coneSpindleRotation} after every time period. Hence, the rotation of the cone in the last \(\delta t\) time units is given by the difference between \text{coneSpindleRotation} and \text{coneRotation}. Since this is the angular displacement of the cone during one time unit, it is also the average speed of the cone in that time unit.

An Eden procedure \text{clock()} is used to model this regular work. The change in rotation is calculated, then the rotations are synchronised and the final work is to place a call to \text{clock()} back onto the stack. This (todo) statement ensures that the next clock cycle happens soon. This is vague enough for our purposes, the length of the time interval \(\delta t\) is irrelevant and can even vary from clock cycle to clock cycle as long as it stays sufficiently small.

Clearly, there needs to be activity taking place during each clock cycle that controls the changes in the wheel’s position. We recall the use of the variable \text{wheelPos} in Section 17.2.5. If \text{wheelPos} is zero then the wheel is at the pointed end of the cone and doesn’t rotate at all. However, as \text{wheelPos} grows, the wheel rolls proportionally more for a given
rotation of the cone. Hence, some code is added to `clock()` to calculate the change in the wheel’s angular displacement, and to accumulate this rotation.

Also added to this procedure is a boolean condition `clockGoing` which allows graceful termination of the clock without the use of the interrupt button in TkEden.

```tcl
proc clock {
    ## Cone speed is the change in angular displacement in the last time unit
    coneSpeed = coneSpindleRotation-coneRotation; ## rads / time
    ## coneRotation is accumulated
    coneRotation = coneRotation+coneSpeed;
    ## wheelSpeed is defined in terms of the coneSpeed
    ## And is the change in angular displacement during the last time unit
    wheelSpeed = -(wheelPos*coneSpeed);
    ## wheelRotation is accumulated
    wheelRotation = wheelRotation+wheelSpeed;
    ## Re-run this procedure soon if not stopped.
    if (clockGoing == 1) {
        todo("clock();");
    }
};
```

### 17.4 Input

The last addition is to add an input window and to link mouse movements in this window to the movement of the tracing pen.

```tcl
%eden
wheelPos is drawPanel_mousePos[2];
carriagePos is -8+16*drawPanel_mousePos[1];
```

### 17.5 Running the model

The model can be loaded by executing the `Run.e` Eden script from the model directory.

```bash
$ tkeden Run.e
```
To enable the rolling, the clock needs to be started.

```plaintext
%eden
clock();
```

A screen shot of the final model is given in Figure 17.7. Movement in the window labelled “screen” will cause movement of the 3D representation in the window labelled “sasami”.

![Figure 17.7: The final model](image-url)
Chapter 18

Distributed Framework

Separating the different elements of the model was so successful in the 2D model (see Section 16) I decided that it should be constructed in a more general context. Planimeter models could then be constructed for this framework that make use of a common input table. There would also be the facility for one input table to drive a number of planimeter models and so allow comparisons of the different mechanisms.

18.1 dTkEden

dTkEden stands for distributed TkEden and provides networkable support for models. A dTkEden window is identified as an agent and has a defined way of interacting with the rest of the model. An agent has a list of global variables that it monitors, known as oracles, and a list of global variables that it has permission to update, known as handles. One dTkEden session runs in server mode and is responsible for receiving updated state from agents with handles and ensuring that agents with oracles are delivered information about relevant state change.

18.2 Structure of the framework

The planimeter framework is a fairly straight-forward dTkEden set up because there are only two types of agents – input table and planimeter. The communication model is
simple since *input table* only has handles on variables and *planimeter* only has oracles.

- The input table provides the user with an interface for moving the pen and manipulates global state (on the server) when the position of the pen changes (or the scales used on the table are modified).

- The planimeter model responds to the motion of the pen (sent to it by the server) and provides the user with a visualisation.

![Figure 18.1: Communication in the framework is one-way](image)

The background to this framework was the work done in the 2D model (See Chapter 16) where there was provision for two input tables and two planimeters. I found that the most useful configuration was one input table with two planimeters connected to it. For this reason, this version and all subsequent versions of the framework were developed with one input table. This modification made it easier to implement the scaling factors that allow the same input table to represent different scales of quantity.

### 18.3 A generic input table

The main benefit of the framework is to allow one input interface to be used on all the planimeters. Because the input table can be re-used, it was justifiable to allocate some
time to developing it further. In the previous models, dividing the user’s attention across two monitors was identified as being a problem. Actions on the input table were causing the planimeter to move when there should been feedback in the input table. The improved input table provides some feedback by having an on-screen cursor showing the position of the tracing pointer.

It was also noticed that it was often helpful for users to investigate the model while someone else was drawing for them. The new input table provides support for automatic drawing so that the computer can automatically draw simple shapes like the unit square.

To ease tracing, grid lines were added to the input table.

18.4 Modification of the cone planimeter

The cone planimeter described in the last chapter was re-written for this framework so that it used the standard variable names for the interface. This lead to the development
of a standard control panel that could also be re-used in other planimeter models.

Figure 18.3: The Oppikofer model modified for the framework

18.5 A generic control panel

The control panel was intended to bring together the best parts of the interfaces so far. They are based on those of the 2D planimeter model, and include controls for the clock, the reset button and a sideways view of the integrating wheel.

18.5.1 Clock control

There had to be a clock control and in previous designs this had been provided by two buttons start and stop. This had bad HCI considerations since a user could press either button at any time. In the new design there is one button that switches its function depending on the status of the clock.
18.5.2 Revolution/area counters

The end-view of the integrating wheel which was a successful component of the 2D model continues to help the user monitor the rotation of the wheel.

The representation of the area has however quite changed. The revolution counter for the disc (that was of no real use) has been removed to reduce confusion. As before, the reset button resets the numerical counters but not the wheel’s physical position.

The new addition to the controls is the second counter that gives the area traced in units. The size of an area unit is configurable by the user by pressing the calibrate button. Pressing this button is as if to say “the area represented by the current rotation of the wheel (since the last zero point) is equal to the unit square”.

![Figure 18.4: The new control panel](image)

18.6 Extending the framework

An obvious application of this framework would be to allow the construction of a differential analyser model. For this to be possible, three extensions or adaptions would need to be made.

1. The provision for numerous input tables would have to be re-introduced. This is a fairly trivial addition.

2. An output table that was capable of plotting a function would need to be modelled.
3. The planimeter models would have to be developed so that they can output data. This would require more complex models.

For more about these and other possible extensions to the project see Section 21.5.
Chapter 19

Modelling a Wetli Planimeter

The second planimeter to be built in the framework was of the Wetli-Starke type. In this sense this model is very much a refined version of the wheel-and-disc planimeter models that were referred to earlier (Chapters 15 and 16). One criticism of these earlier planimeters was that they just showed the wheel and disc rotating in the abstract without the context of their guiding rails, carriages or shafts. This model tries to address those issues by merging the success of the cone planimeter model with the original work undertaken.

Like the cone model, this planimeter was designed as a 3D model from scratch. It makes use of the Eden function `macro()`\(^1\) that allows common code to be reproduced by variable substitution.\(^2\) This function was used to generate all the cuboids in the model and makes the code much easier to understand.

### 19.1 Using the macro function

To generate an object in Sasami; all vertices and surfaces need to be defined separately. A section of Sasami script to define a cube might therefore be the declarations of the 8 vertices.

\(^1\) `macro()` is missing from the Eden Handbook and really should not be. It is a very useful tool particularly for Sasami when huge amounts of code have to be reproduced to copy something simple like a cube.

\(^2\) `macro()` reads a character string and replaces occurrences of ?1, ?2, etc. with character strings that have been passed to it.
19.2 Constructing the model

This model took a slightly different approach. Starting from a rough sketch, key dimensions were represented by Eden variables. From there other dimensions were defined with dependency allowing cuboids defined by macros to be sized correctly and placed at appropriate places. This provided the model with a great flexibility of interaction, one example of which is shown below.
19.3 Manipulating the model

Being designed around dimensions allows some interesting modifications to be made to the model at run-time. For example, we could perhaps investigate what would happen if the track were extended.

```
## Set the dimension trackLength to 5 times its previous value
%eden
trackLength=trackLength*5;
```

Changing a dimension like this causes a number of vertices and polygons to be re-defined as the track is stretched (see Figure 19.2).

19.4 Running the model

This model is included in the framework directory as planimeter1.
Figure 19.2: The model with a stretched track
Chapter 20

Modelling a Amsler Planimeter

The model of an Amsler planimeter was a late addition to the models produced and was actually written to assist my understanding of their mechanism. In this sense, the model is a superb advocate for the empirical modelling tools being a way of “thinking with computers”. The diagram is in 2D (in DoNaLD) which was found to be quite sufficient to illustrate this type of mechanism. Another motivation for the model was ‘The Planimeter Applet’ (described Section 13.1) although the model is not as complex as the applet – which uses 1000 lines more code. However, most of the interaction provided by the applet could be incorporated into the Eden model fairly easily.

The model illustrates the two arms of the Amsler planimeter as two lines between three dots. One of these dots represents the tracing point. An end-on visualisation of the wheel is provided to show the rotation and a reset button to reset the counter. In an actual polar planimeter, the wheel is mounted at some point along the arm between the pivot point and the tracing pen.

Unlike the reset button of the earlier wheel-and-disc model (Section 16.4), the reset button on the Amsler model does rotate the wheel to an initial starting point. This is how the planimeters were actually operated. The planimeter was lifted off its surface and the wheel was moved to the zero point.\(^1\)

\(^1\)check reference in [Hen94]
20.1 How to run this model

This model requires no setting up or networking. It is just loaded into TkEden and starts working.

`$ /dcs/emp/empublic/bin/tkeden amslerModel.eden`

The reset button will reset the area counter. Once it has been clicked (and is therefore in focus) it is sensitive to key actions. This provides the means – by hitting any key – to reset the area counter while tracing.
Chapter 21

Conclusion

The original specification of the project is included in Appendix A. The project was highlighted as having three main components; history, modelling and presenting the history.

21.1 History

At the beginning of the project I set myself three objectives for the historical research. I wanted to find out about the different people involved, how their work related to each other and what their motivation was.\textsuperscript{1} Looking at the inventors individually has confirmed that many people did conceive of the idea independently. In terms of motivation, it was the need to measure land area that created the need for planimeters to be invented. It has been interesting to look at how many of the early planimeters were brought together at the Great Exhibition. It would be interesting to look at planimeters displayed at other European exhibitions of the same period. It is fascinating that Sang’s work was so independent of the European work and one day I would like to be able to answer the question of how much Maxwell or Thomson’s innovation was isolated from Europe.

\textsuperscript{1}See Section A.1
21.2 Modelling

Producing models of historical devices is a great challenge. There is a tension between historical accuracy and the illustration of principle. In my models the emphasis has usually been on providing users with a tool to understand the principle of a planimeter rather than just being a digital exhibit.

The project has been successful in demonstrating that an analogue device can be modelled on a digital platform. Using tools that support dependency was definitely the correct choice, I am very pleased that most of the models have analogue principles encoded in the deepest elements of their behaviour. In this way the models really are analogue in ‘thought and word and deed’.

21.3 Illustrating history

The ‘presentation’ section of the project has received the least treatment. It was hoped that “some kind of hypermedia”\(^2\) would be produced. Unfortunately, this area of work did not fit into the time constraints of the project.

My planimeter models have had a great deal of exposure to a variety of audiences. As well as the project presentation, the models have been used on an open day and shown to interested academics. Their value as a teaching aid should not be underestimated. When exhibited at a department open day, the models were being used by people who had never even heard of planimeters before.

21.4 Project management

21.4.1 Timetable

Section A.6 shows the original timetable for the planned work. The flow of work outlined here was adhered to without much alteration. Forcing myself to produce extra documen-

\(^2\)See Section A.2
21.5. FURTHER WORK

I found that tasks based around writing tended to require more hours than expected whereas – unusually for Computer Science – the coding actually took less. The research element of the project consumed as much time as was allocated although I was extremely pleased with the results that have been achieved. Preparation for the presentation ran to schedule.

21.4.2 Writing

The discipline of producing structured writing early on in the project provided some basic content for starting the report-writing phase. Using \LaTeX{} for all documentation (including the presentation slides) enabled me to develop proficiency with both \LaTeX{} and \BibTeX{} and to have the confidence to use these tools in my report.

21.4.3 Website

All finished documentation was published on my project web pages. This helped me visibly track the project’s progress and provided something for interested parties to explore. Screenshots from my models were also added to the site giving a complete picture of the work going on.

21.5 Further work

21.5.1 Further integration of history and models

As they stand, the models need some kind of background explanation. Further work should be done with the models to allow them to be presented alongside some form of hypertext. Earlier it was suggested that a browser ‘plug-in’ version of TkEden could be developed.

\footnote{See Section A.4.}
21.5.2 Modelling the differential analyser

The background to investigating the history of the planimeter was the use of wheel-and-disc integrators in the differential analyser. A logical extension to the project would be to connect planimeters together and model a differential analyser.

21.5.3 The development of the polar planimeter

The widely manufactured polar planimeter was only touched on by this project. Further research should be done to access the originality of the various developments relating to the original work of Amsler. Models of rolling planimeters would be an interesting goal to work towards.
Appendix A

Project Specification

Illustrating the History of the Planimeter
Charles Care
17th October 2003

A.1 Problem

“One most common form of mathematical instrument available in the
nineteenth century was the planimeter.”

An analogue device that can perform mathematical integration, the planimeter was
firstly invented in Bavaria in 1814. Throughout the 19th century it was developed in
a variety of ways, receiving many of its improvements from famous scientific names such
Lord Kelvin and James Maxwell. Later it would become a central component of famous
machines like Kelvin’s Harmonic Analyser and the Differential Analysers of the early 20th
century.

The purpose of the project is to trace the history of this significant, but often forgotten
device; and to illustrate and present the story of its development with a number of
computer-based graphical models. To this end, three main components of the project can
be identified.

- History – Particularly of interest are the people involved, how their work related to
each other, and what the motivation for their work was.
- Modelling – To produce some simulation of the devices researched.
- Presentation – It is hoped that the research and the models can be linked together
to provide an accessible way of exploring the history.

1(Mary Croarken: Early Scientific Computing in Britain. Oxford 1990.)
4(William Aspray: Computing before Computers.)
A.2 Objectives

The project will involve work in the following areas:

1. Research into the history of the planimeter.
   - Learning how to find and use historical sources.
   - Learning how to write-up and cite historical research.
   - Representing the history in an accessible way, perhaps as some form of hypermedia.

2. How to model an analogue device.
   - Evaluation of potential modelling methods/tools.
   - Producing flexible models that illustrate the development history of the planimeter.

A.3 Methods

It is planned that research will continue throughout the project in parallel with the modelling process. A number of milestones are defined in the project timetable which will synchronise these two separate areas of work from time to time.

Regular project reports will be produced, to keep track of current progress.

The project can be split into a number of tasks. These are laid out in a Gantt Chart that appears as an appendix to this document.

A.3.1 Preparation for Modelling

The following tasks are identified as being necessary before work on the final models begin.

- Learning Basic Eden & Associated Tools – It is planned to spend a few weeks working on some example code and understanding the Empirical Modelling Paradigm.

- Java 3D – Some time will be spent learning the basics of Java 3D.

- JaM API – Some time will be spent becoming more familiar with this way of using dependencies in a Java context.

- Language Evaluation – Once a working knowledge of both the Eden-based tools and the JaM framework has been acquired, a basic model of the planimeter will be attempted in both languages. Although these models may not reach a finished state, it should be clear after some initial work which environment will offer the best trade-off between presentational quality and flexibility of design. The findings of this evaluation will be presented in a report.
A.3. METHODS

A.3.2 The Modelling Process

This will begin in week 9 and will draw on the research undertaken in term 1. It is possible that further research might adjust the way the model is structured and any new material will be merged at the beginning of term 2.

Models will be built in the language proposed by the preparatory work.

A.3.3 The Research Process

The research process can be broken down into two main sections, preliminary research and the preparation of some research reports. The preliminary work will be completed within a few weeks and the two initial documents a few weeks later. Once completed, these two reports will form a basis to decide which areas the project should focus on.

Preliminaries

Some work has already been undertaken during the summer to locate some basic sources for the historical element of the project. Brief notes on these sources can be found in the following project documents.

- project-notes-001
- project-notes-002
- project-notes-003
- project-notes-004
- project-notes-005
- project-notes-006

Research Documents

Two documents will be produced during the first term to summarise the research undertaken. These reports will form the basis for deciding what the research focus should be for the rest of the project.

1. A summary of the inventors.

2. A summary of the types of planimeter.
APPENDIX A. PROJECT SPECIFICATION

A.4 Timetabled Milestones

1. Term 1 Week 5: Summary of Inventors.
2. Term 1 Week 7: Modelling Language Report
3. Term 1 Week 8: Summary of Devices.
4. Term 1 Week 10: End of Term Progress Report

A.5 Resources

The first part of the Project is mainly research based and so the use of library facilities (both internal and external) will be employed. For the modelling it is planned that the bulk of work should be undertaken locally on the dcs machines using either tkeden or the Java JaM API.

It is possible that some work will be undertaken remotely. The intention is that any remote work will be synchronised and if necessary merged with the data on dcs servers as often as practical.

The project is intended to be dependent on campus-based computing so that any personal facilities used are non-essential.

A.5.1 Report Writing Tools Identified

- Basic Unix
- L\textsc{\textup{a}}\textsc{t}eX
- Perl

A.5.2 Modelling Tools Identified

- Java SDK
- TkEden – Warwick Empirical Modelling
- DoNaLD – Warwick Empirical Modelling
- Sasami – Warwick Empirical Modelling
- JaM – Warwick Empirical Modelling
Appendix B

CD Contents

The following directories are available on the CD. The models need the eden tools \texttt{dtkeden} and \texttt{tkeden} in \texttt{/dcs/emp/emppublic/bin/}.

- \textbf{code} – Source code for the models
  - model1 – See Chapter 15
  - model2 – See Chapter 16
  - OppikoferModel – See Chapter 17
  - framework – See Chapters 18 and 19
  - MacroDemo.eden – See Chapter 19
  - amslerModel – See Chapter 20

- \textbf{report} – pdf file of report.
Bibliography

[Anoa] Diagram of Clair’s planimeter from the Science Museum.


[GE52] *Exhibition of the Works of Industry of all Nations, 1851 : reports by the juries on the subjects in the thirty classes into which the exhibition was divided*. Clowes, London, 1852.


